International Journal of Thermophysics, Vol. 27, No. 3, May 2006 (© 2006)
DOI: 10.1007/s10765-006-0048-z

Reflection and Refraction of Micropolar Magneto-thermoviscoelastic Waves at the Interface between Two Micropolar Viscoelastic Media

Yaqin Song,[1,2] Hongyu Xu,[3] and Yuanchong Zhang[1]

Received November 8, 2004

Using micropolar generalized thermoviscoelastic theories, problems of reflection and refraction of magneto-thermoeviscoelastic waves at the interface between two viscoelastic media are studied when a uniform magnetic field permeates the media. Coefficient ratios of reflection and refraction are obtained using continuous boundary conditions. Some special cases are considered, i.e., the absence of micropolar and viscous effects. By numerical calculations, variations of the amplitude ratios of reflection and refraction coefficients with the angle of incidence are shown graphically for incident rotational and dilatational waves at the interface between two media (one medium is aluminium-epoxy micropolar viscoelastic material, and the other is magnesium crystal micropolar viscoelastic material). Comparing the generalized thermoelastic theories developed by Lord and Shulman (LS) and by Green and Lindsay (GL) in this paper to conventional dynamics (CD) theory the effects of a magnetic field and viscosity are shown numerically in this paper.

KEY WORDS: generalized thermoelastic theory; magneto-thermoviscoelastic waves; reflection and refraction; relaxation times.

1. INTRODUCTION

The heat conduction equations for classical uncoupled and coupled theories of thermoelasticity (here called conventional dynamics, or CD, theory)

[1] School of Aerospace, Department of Engineering Mechanics, MOE Key Laboratory for Strength and Vibration, Xi'an Jiaotong University, Xi'an 710049, P. R. China.
[2] To whom correspondence should be addressed. E-mail: yqsong@mail.xjtu.edu.cn
[3] School of Architecture Engineering, Henan University of Science and Technology, Luoyang 471003, P. R. China.

are of the diffusion type, and predict an infinite speed of propagation of the heat wave, which is physically inadmissible. To eliminate this paradox of the classical approach, theories of generalized thermoelasticity were developed. At present, there are various generalized approaches, but the theories proposed by Lord and Shulman [1] and Green and Lindsay [2] (here called LS and GL theories, respectively) are most popular. These theories have been developed by introducing one or two relaxation times in the thermoelastic process, with an aim to eliminate the paradox of an infinite speed for the propagation of thermal signals. The LS model is based on a modified Fourier's law, but the GL model even allows second sound without violating the classical Fourier's law. The two theories are structurally different, and one cannot be obtained as a particular case of the other. Various problems characterizing these two theories have been investigated and have revealed some interesting phenomena. Chandrasekharaiah [3,4] has reported brief reviews of this topic.

The linear micropolar theory has been developed by Eringen [5]. It is used to describe polymers and materials possessing microstructures. Metals, polymers, composites, soils, rocks, and concretes are typical media with microstructures. The difference between a micropolar theory and classical theory is the introduction of an independent microrotation vector. Thus, the rotation of the continuum includes macrorotation and microrotation. Also, there exists not only a traditional stress tensor but also a coupled stress tensor. And the stress tensor can be asymmetrical when the rotation of a microvolume is in equilibrium. In recent years, the study of micropolar theory has become more important due to the large-scale exploitation and application of composites, polymers, and large-grain materials. At the present time, many studies have been reported for the micropolar theory under generalized thermoelastic and thermoviscoelasticity theories [6–13].

The problem of the reflection and refraction of plane waves at a plane interface in a micropolar medium has been discussed by many authors, e.g., Parfitt and Eringen [14], Ariman [15], etc. Recently, Singh and Kumar [16,17] investigated the reflection and refraction of an interface wave between a viscoelastic solid and a micropolar elastic solid. Kumar [18] studied wave propagation in a micropolar viscoelastic generalized thermoelastic solid. Kumar and Deswal [19] studied the propagation of a surface wave in micropolar thermoelastic materials under thermoelasticity without energy dissipation. But few papers have been concerned with problems of wave propagation at the interface between two micropolar viscoelastic media under a permeating magnetic field in generalized thermoviscoelastic theory.

In this paper we studied reflection and refraction of such interfaces in a magnetic field within the framework of generalized thermoviscoelasticity. Equations for the reflection and refraction coefficient ratios of dilatational and rotational waves are given using continuity boundary conditions at the interface. Two special cases, (a) absence of micropolar effect and (b) absence of viscous effect, are considered. Variations of the amplitude ratios of the reflection and refraction coefficients with the angle of incidence for aluminum-epoxy and magnesium crystal micropolar viscoelastic materials are presented based on numerical calculations. Also, comparisons among CD, LS, and GL theories, and the effects of viscous and magnetic fields are shown graphically.

2. FORMULATION OF PROBLEM

Consider isotropic, homogeneous, linear, thermally and electrically conducting micropolar viscoelastic media, M_1 and M_2, occupying the semi-infinite Cartesian space: $\Gamma_1 = \{(x, y, z) \,|-\infty < x, y < \infty, -\infty < z \le 0\}$ and $\Gamma_2 = \{(x, y, z) \,|-\infty < x, y < \infty, 0 < z \le \infty\}$, respectively. The whole body is at a constant temperature T_0, and it is acted on throughout by a constant magnetic field $\vec{\mathbf{H}} = (0, H_0, 0)$, which is oriented towards the positive direction of the y-axis. Assume the components of the displacement and microrotation are $\vec{\mathbf{u}} = (u, 0, w)$ and $\vec{\boldsymbol{\omega}} = (0, \omega_2, 0)$.

The governing equations of the problem follow.

(a) Equation of motion (in the absence of body force and heat source):

$$\rho \ddot{u}_i = \sigma_{ij,j} + \vec{\mathbf{F}}_i, \tag{1}$$

$$\in_{ijp} \sigma_{jp} + m_{ji,j} = J\rho \frac{\partial^2 \omega_i}{\partial t^2}, \tag{2}$$

where the Lorentz force is given by

$$\vec{\mathbf{F}} = \mu_0 \left(\vec{\mathbf{J}} \times \vec{\mathbf{H}} \right); \tag{3}$$

$\in_{ijp}$ is the alternating tensor and defined as

$$\in_{ijp} = \mathbf{n}_i \cdot \left(\mathbf{n}_j \times \mathbf{n}_p \right). \tag{4}$$

The variations of the magnetic and electric fields are given by Maxwell equations:

$$\mathrm{Curl}\, \vec{\mathbf{H}} = \vec{\mathbf{J}} + \varepsilon_0 \dot{\vec{E}}, \tag{5}$$

$$\mathrm{curl}\, \vec{\mathbf{E}} = -\mu_0 \dot{\vec{\mathbf{h}}}, \tag{6}$$

$$\vec{\mathbf{E}} = -\mu_0 \left(\dot{\vec{\mathbf{u}}} \times \vec{\mathbf{H}}\right), \tag{7}$$

$$\operatorname{div} \vec{\mathbf{h}} = 0, \tag{8}$$

And the current density $\vec{\mathbf{J}}$ is obtained by Ohm's law,

$$\vec{\mathbf{J}} = \sigma_0 \left(\vec{\mathbf{E}} + \frac{\partial \vec{\mathbf{u}}}{\partial t} \times \vec{\mathbf{B}}\right). \tag{9}$$

The components of the initial magnetic field can be written as

$$H_x = H_z = 0, \quad H_y = \mu_0 H_0 = B_0. \tag{10}$$

The components of the Lorentz force are given by

$$F_x = -\sigma_0 B_0^2 \frac{\partial u}{\partial t}, \quad F_y = 0, \quad F_z = -\sigma_0 B_0^2 \frac{\partial w}{\partial t}. \tag{11}$$

(b) Constitutive equation
Assuming that the relaxation effects of the volumetric properties of the material are ignored, one can write for the generalized theory of thermo-viscoelasticity,

$$\begin{aligned}\sigma_{ij} = \widehat{R}\left(e_{ij}\right) + \left(Ke - \frac{\widehat{R}\,(e)}{3}\right)\delta_{ij} + k\left(u_{j,i} - \epsilon_{ijp}\omega_p\right) \\ -(3\lambda + 2\mu + k)\alpha_t \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T\delta_{ij}.\end{aligned} \tag{12}$$

where

$$\begin{aligned}&\int_0^t R(t-\tau)\frac{\partial \varepsilon_{ij}(\vec{\mathbf{x}}, \tau)}{\partial \tau} d\tau = \widehat{R}(\varepsilon_{ij}), \quad e = e_{ii}, \quad e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \\ &\omega_{ij} = \frac{1}{2}\left(\omega_{i,j} + \omega_{j,i}\right), \quad K = \lambda + \frac{2}{3}\mu.\end{aligned} \tag{13}$$

δ_{ij} is the Kronecker delta, defined as $\delta_{ij} = \mathbf{n}_i \cdot \mathbf{n}_j$. The elastic case can be obtained when
$\widehat{R}\left(\varepsilon_{ij}\right) = 2\mu\left(\varepsilon_{ij}\right)$.
$R\,(t)$ is a relaxation function given by [20]

$$R\,(t) = 2\mu\left[1 - A\int_0^t e^{-\beta t} t^{\alpha' - 1} dt\right], \tag{14}$$

where $(0 < \alpha' < 1, A > 0, \beta > 0)$ are experimental parameters.
The relation between the coupled stress and rotation tensor is given below:

$$m_{ij} = \alpha\omega_{ii}\delta_{ij} + \beta\omega_{i,j} + \gamma\omega_{j,i} \tag{15}$$

(c) Generalized heat conduction equation

$$\begin{aligned} k'\nabla^2 T &= \rho C_E \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right) T \\ &\quad + (3\lambda + 2\mu + k)\,\alpha_t T_0 \left(\frac{\partial}{\partial t} + n\tau_0 \frac{\partial^2}{\partial t^2}\right) e_{kk}, \end{aligned} \tag{16}$$

The use of relaxation times τ_0, τ_1 and dimensionless constant n makes the above equations possible for the three different theories:

CD theory:	$\tau_0 = \tau_1 = 0$
LS theory:	$\tau_0 > 0, \quad \tau_1 = 0, \quad n = 1$
GL theory:	$\tau_1 \geqslant \tau_0 \geqslant 0, \quad n = 0$

Using Eq. (1), (2), (11), and (12), we obtain the following equations of motion:

$$\begin{aligned} &\left(K + \frac{\widehat{R}}{6}\right)\frac{\partial^2 w}{\partial x \partial z} + \left(K + k + \frac{2\widehat{R}}{3}\right)\frac{\partial^2 u}{\partial x^2} + \left(k + \frac{\widehat{R}}{2}\right)\frac{\partial^2 u}{\partial z^2} - k\frac{\partial \omega_2}{\partial z} \\ &\quad -(3\lambda + 2\mu + k)\alpha_t\left(1 + \tau_1\frac{\partial}{\partial t}\right)T_{,x} + \mu_0(\vec{J}\times\vec{H})_x = \rho\,\ddot{u}, \end{aligned} \tag{17}$$

$$\begin{aligned} &\left(K + \frac{\widehat{R}}{6}\right)\frac{\partial^2 u}{\partial x \partial z} + \left(K + k + \frac{2\widehat{R}}{3}\right)\frac{\partial^2 w}{\partial z^2} + \left(k + \frac{\widehat{R}}{2}\right)\frac{\partial^2 w}{\partial x^2} + k\frac{\partial \omega_2}{\partial x} \\ &\quad -(3\lambda + 2\mu + k)\alpha_t\left(1 + \tau_1\frac{\partial}{\partial t}\right)T_{,z} + \mu_0(\vec{J}\times\vec{H})_z = \rho\,\ddot{w}, \end{aligned} \tag{18}$$

$$(\alpha + \beta)\omega_{k,ki} + \gamma\omega_{i,kk} - k\epsilon_{ijk}u_{j,k} - 2k\omega_i = \rho J\ddot{\omega}_i. \tag{19}$$

To obtain a solution of the problem, the following displacement potentials ϕ and $\vec{\psi} = (0, \psi, 0)$ and microrotation potentials ξ and $\vec{\zeta} = (\zeta_1, 0, \zeta_3)$ are introduced using the following relations:

$$\vec{\boldsymbol{u}} = \nabla\phi + \nabla\times\vec{\boldsymbol{\psi}}, \qquad \nabla\cdot\vec{\boldsymbol{\psi}} = 0, \\ \vec{\boldsymbol{\omega}}_2 = \nabla\xi + \nabla\times\vec{\boldsymbol{\zeta}}, \qquad \nabla\cdot\vec{\boldsymbol{\zeta}} = 0 \quad . \tag{20}$$

From Eqs. (16)–(20), we get

$$\mathrm{k}'\nabla^2\mathrm{T} = \rho\mathrm{C_E}\left(\frac{\partial}{\partial \mathrm{t}} + \tau_0\frac{\partial^2}{\partial \mathrm{t}^2}\right)\mathrm{T} \\ + (3\lambda+2\mu+\mathrm{k})\,\alpha_\mathrm{t}\mathrm{T}_0\left(\frac{\partial}{\partial \mathrm{t}} + \mathrm{n}\tau_0\frac{\partial^2}{\partial \mathrm{t}^2}\right)\nabla^2\phi, \tag{21}$$

$$\alpha_1\frac{\partial^2\phi}{\partial \mathrm{t}^2} = \left(\frac{\mathrm{K}+\mathrm{k}}{\rho} + \frac{\mathrm{KR_H}}{\rho} + \frac{2\widehat{\mathrm{R}}}{3\rho}\right)\nabla^2\phi \\ - \frac{(3\lambda+2\mu+\mathrm{k})\alpha_\mathrm{t}}{\rho}\left(1+\tau_1\frac{\partial}{\partial \mathrm{t}}\right)\mathrm{T}, \tag{22}$$

$$\alpha_1\frac{\partial^2\vec{\boldsymbol{\psi}}}{\partial \mathrm{t}^2} = \frac{\widehat{\mathrm{R}}}{2\rho}\nabla^2\vec{\boldsymbol{\psi}} + \frac{\mathrm{k}}{\rho}\nabla\times\vec{\boldsymbol{\zeta}}, \tag{23}$$

$$\frac{\partial^2\vec{\boldsymbol{\zeta}}}{\partial \mathrm{t}^2} = \frac{\gamma}{\rho\mathrm{J}}\nabla^2\vec{\boldsymbol{\zeta}} - 2\omega_0^2\vec{\boldsymbol{\zeta}} + \omega_0^2\nabla\times\vec{\boldsymbol{\psi}}, \tag{24}$$

$$\frac{\partial^2\xi}{\partial t^2} = \frac{\gamma}{\rho\mathbf{J}}\nabla^2\xi - 2\omega_0^2\xi. \tag{25}$$

where

$$\alpha_1 = 1 + \frac{\mathrm{C_A^2}}{\mathrm{c}^2}, \quad \mathrm{c}^2 = \frac{1}{\varepsilon_0\mu_0}, \quad \mathrm{C_A^2} = \frac{\mu_0\mathrm{H}_0^2}{\rho}, \\ \mathrm{C_T} = \sqrt{\frac{\mathrm{K}}{\rho}},\ \mathrm{R_H} = \frac{\mathrm{C_A^2}}{\mathrm{C_T^2}}, \quad \omega_0^2 = \frac{\mathrm{k}}{\rho\mathrm{J}}, \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}. \tag{26}$$

From Eqs (21)–(25), we notice that while the dilatational displacement wave is affected due to the presence of the thermal and magnetic effects, the dilatational wave, the coupled rotational displacement wave, and the microrotational waves remain unaffected.

Expressions for the stress components are

$$\sigma_{zz}=\left(K-\frac{\widehat{R}}{3}\right)\nabla^2\phi+\left(k+\widehat{R}\right)\left(\frac{\partial^2\phi}{\partial z^2}-\frac{\partial^2\psi}{\partial x\partial z}\right)-\gamma_1\left(1+\tau_1\frac{\partial}{\partial t}\right)T, \quad (27)$$

$$\sigma_{xz}=\left(k+\widehat{R}\right)\frac{\partial^2\phi}{\partial x\partial z}+\left(k+\frac{\widehat{R}}{2}\right)\frac{\partial^2\psi}{\partial x^2}-\frac{\widehat{R}}{2}\frac{\partial^2\psi}{\partial z^2}+k\omega_2, \quad (28)$$

$$\sigma_{zx}=\left(k+\widehat{R}\right)\frac{\partial^2\phi}{\partial x\partial z}-\left(k+\frac{\widehat{R}}{2}\right)\frac{\partial^2\psi}{\partial z^2}+\frac{\widehat{R}}{2}\frac{\partial^2\psi}{\partial x^2}-k\omega_2, \quad (29)$$

$$\sigma_{xx}=\left(K-\frac{\widehat{R}}{3}\right)\nabla^2\phi+\left(k+\widehat{R}\right)\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\psi}{\partial x\partial z}\right)-\gamma_1\left(1+\tau_1\frac{\partial}{\partial t}\right)T, \quad (30)$$

$$m_{zy}=\gamma\frac{\partial\omega_2}{\partial z} \quad (31)$$

where

$$\gamma_1=(3\lambda+2\mu+k)\,\alpha_t.$$

3. REFLECTION AND REFRACTION OF WAVES

Assuming the motion is harmonic, we give solutions of the problem in the following form:

$$\{\phi,\psi,T,\omega_2\}=\{\phi_1,\psi_1,T_1,\omega_{21}\}\exp(-i\omega t), \quad (32)$$

Substituting the above expression into Eqs (21)–(24), we get

$$\left(\nabla^4+N_1\nabla^2+N_2\right)\phi_1=0 \quad (33)$$

$$\left(\nabla^4+N_3\nabla^2+N_4\right)\psi_1=0 \quad (34)$$

in which

$$\overline{R}=-\frac{2\mu}{i\omega}\left[1-\frac{A\Gamma\left(\alpha'\right)}{(\beta-i\omega)^{\alpha'}}\right],$$

$$N_1=\frac{k'\alpha_1\omega^2-\xi_1\xi_3-\xi_2\xi_4}{k'\xi_1},$$

$$N_2 = -\frac{\alpha_1 \omega^2 \xi_3}{k' \xi_1},$$
$$N_3 = \frac{\left(i\omega\overline{R} + 2k\right) \rho J \left(\omega^2 - 2\omega_0^2\right) + 2\rho k J \omega_0^2 + 2\alpha_1 \rho \omega^2 \gamma}{\left(i\omega\overline{R} + 2k\right) \gamma}, \tag{35}$$

$$N_4 = \frac{2\alpha_1 \rho^2 J \omega^2 \left(\omega^2 - 2\omega_0^2\right)}{\left(i\omega\overline{R} + 2k\right) \gamma}, \quad \xi_1 = \frac{K + k + \mu_0 H_0^2 - 2i\omega\overline{R}/3}{\rho}, \quad \xi_2 = \frac{\gamma_1 \bar{\tau}_1}{\rho},$$
$$\xi_3 = -i\omega\rho C_E \bar{\tau}_0, \quad \xi_4 = -i\omega\gamma_1 T_0 \bar{\tau}_n, \quad \bar{\tau}_{0,1} = 1 - i\omega\tau_{0,1}, \quad \bar{\tau}_n = 1 - in\omega\tau_0,$$

We can see from Eqs (33) and (34) that there exist four waves propagating with different velocities at the interface between two semi-infinite micropolar viscoelastic media; two of them are dilatational waves, and the other two are rotational waves. The velocities of the rotational waves are

$$v_{1,2}^2 = \frac{\omega^2 \left(N_1 \pm \sqrt{N_1^2 - 4N_2}\right)}{2N_2}; \tag{36}$$

the velocities of the dilatational waves are

$$v_{3,4}^2 = \frac{\omega^2 \left(N_3 \pm \sqrt{N_3^2 - 4N_4}\right)}{2N_4} \tag{37}$$

From the above we can see there are four reflected waves and four refracted waves (two of them are dilatational waves, the other two are rotational waves) when waves from inside of one medium arrive at the interface of the two media where z=0. Assume that the propagating direction of the incident wave make an angle θ with the positive direction of the z-axis. The propagating directions of the reflected and refracted waves make angles $\theta_1, \theta_2, \theta_3, \theta_4$ and $\theta_1^*, \theta_2^*, \theta_3^*, \theta_4^*$ with the positive direction of the z-axis, respectively (Fig. 1).

(a) For incident rotational waves, the displacement potentials, ϕ and ψ, have the following forms:

$$\phi = \sum_{i=1}^{2} E_i \exp\left\{ik_i(\cos\theta_i x + \sin\theta_i z) - i\omega_i t\right\} \tag{38}$$

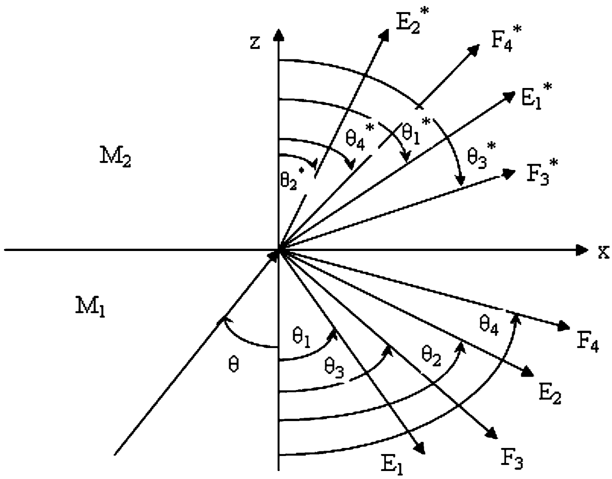

Fig. 1. Reflection and refraction of plane waves at the interface between two micropolar viscoelastic media.

$$\psi = F_1 \exp\left\{ik_0(\cos\theta_0 x - \sin\theta_0 z) - i\omega_0 t\right\} + \sum_{i=3}^{4} F_i \exp\left\{ik_i(\cos\theta_i x + \sin\theta_i z) - i\omega_i t\right\} \tag{39}$$

$$T = \sum_{i=1}^{2} \beta_i E_i \exp\left\{ik_i(\cos\theta_i x + \sin\theta_i z) - i\omega_i t\right\} \tag{40}$$

$$\omega_2 = \beta_3 F_1 \exp\left\{ik_0(\cos\theta_0 x - \sin\theta_0 z) - i\omega_0 t\right\} + \sum_{i=3}^{4} \beta_i F_i \exp\left\{ik_i(\cos\theta_i x + \sin\theta_i z) - i\omega_i t\right\} \tag{41}$$

where $E_1, E_2, F_1, F_3, F_4, \beta_1, \beta_2, \beta_3, \beta_4$ are complex constants, and

$$\begin{aligned} \beta_j &= \frac{\alpha_1\omega_j^2 - \xi_1 k_j^2}{\xi_2}, \qquad (j = 1, 2) \\ \beta_j &= -\frac{2\alpha_1\rho\omega_j^2 + \left(-i\omega_j\overline{R} + 2k\right)k_j^2}{2k}, \qquad (j = 3, 4), \end{aligned} \tag{42}$$

(b) For incident dilatational waves, the displacement potentials, ϕ and ψ, have the following forms:

$$\phi = F_1 \exp\left\{ik_0(\cos\theta_0 x - \sin\theta_0 z) - i\omega_0 t\right\} + \sum_{i=1}^{2} E_i \exp\left\{ik_i(\cos\theta_i x + \sin\theta_i z) - i\omega_i t\right\} \tag{43}$$

$$\psi = \sum_{i=3}^{4} F_i \exp\{ik_i(\cos\theta_i x + \sin\theta_i z) - i\omega_i t\} \tag{44}$$

$$T = \beta_1 F_1 \exp\left\{ik_0(\cos\theta_0 x - \sin\theta_0 z) - i\omega_0 t\right\} + \sum_{i=1}^{2} \beta_i E_i \exp\left\{ik_i(\cos\theta_i x + \sin\theta_i z) - i\omega_i t\right\} \tag{45}$$

$$\omega_2 = \sum_{i=3}^{4} \beta_i F_i \exp\left\{ik_i(\cos\theta_i x + \sin\theta_i z) - i\omega_i t\right\} \tag{46}$$

The potential functions of the refraction waves in both cases can be written as

$$\phi^* = \sum_{i=1}^{2} E_i^* \exp\left\{ik_i^*(\cos\theta_i^* x - \sin\theta_i^* z) - i\omega_i^* t\right\} \tag{47}$$

$$\psi^* = \sum_{i=3}^{4} F_i^* \exp\left\{ik_i^*(\cos\theta_i^* x - \sin\theta_i^* z) - i\omega_i^* t\right\} \tag{48}$$

$$T^* = \sum_{i=1}^{2} \beta_i^* E_i^* \exp\left\{ik_i^*(\cos\theta_i^* x - \sin\theta_i^* z) - i\omega_i^* t\right\} \tag{49}$$

$$\omega_2^* = \sum_{i=3}^{4} \beta_i^* F_i^* \exp\left\{ik_i^*(\cos\theta_i^* x - \sin\theta_i^* z) - i\omega_i^* t\right\} \tag{50}$$

in which $k_i^*, \theta_i^*, \omega_i^*$ in Eqs (47)–(50) correspond with k_i, θ_i, ω_i in Eqs (38)–(46), and β_i^* corresponds with β_i in Eq. (42). Symbols with an asterisk in Medium M_2 denote the same meaning as those without asterisks in Medium M_1.

4. BOUNDARY CONDITIONS

We consider the following continuity boundary conditions of stress, displacement, and microrotation:

$$\sigma_{zz} = \sigma^{*}_{zz} \qquad \text{for } z = 0. \tag{51}$$

$$\sigma_{zx} = \sigma^{*}_{zx} \qquad \text{for } z = 0. \tag{52}$$

$$m_{zy} = m^{*}_{zy} \qquad \text{for } z = 0. \tag{53}$$

$$u = u^{*} \quad \text{for } z = 0. \tag{54}$$

$$w = w^{*} \quad \text{for } z = 0. \tag{55}$$

$$\omega_2 = \omega^{*}_2 \quad \text{for } z = 0. \tag{56}$$

Also, the temperature and temperature gradient are continuous at the interface:

$$T = T^{*} \quad \text{for } z = 0. \tag{57}$$

$$k' \frac{\partial T}{\partial z} = k'^{*} \frac{\partial T^{*}}{\partial z} \quad \text{for } z = 0. \tag{58}$$

5. REFLECTION AND REFRACTION COEFFICIENT RATIOS

The potentials given in Eqs (38)–(41) and (43)–(50) satisfy the boundary conditions, Eqs (51)–(58), at the interface z=0, if

$$\omega_1 = \omega_2 = \omega_3 = \omega_4 = \omega^{*}_1 = \omega^{*}_2 = \omega^{*}_3 = \omega^{*}_4 = \omega \tag{59}$$

and

$$\begin{aligned} & k_0 \cos\theta_0 = k_1 \cos\theta_1 = k_2 \cos\theta_2 = k_3 \cos\theta_3 = k_4 \cos\theta_4 = k^{*}_1 \cos\theta^{*}_1 \\ & = k^{*}_2 \cos\theta^{*}_2 = k^{*}_3 \cos\theta^{*}_3 = k^{*}_4 \cos\theta^{*}_4 \end{aligned} \tag{60}$$

From Eq. (60) we obtain

$$\frac{\cos\theta_0}{v_0}=\frac{\cos\theta_1}{v_1}=\frac{\cos\theta_2}{v_2}=\frac{\cos\theta_3}{v_3}=\frac{\cos\theta_4}{v_4}=\frac{\cos\theta_1^*}{v_1^*}=\frac{\cos\theta_2^*}{v_2^*}=\frac{\cos\theta_3^*}{v_3^*}=\frac{\cos\theta_4^*}{v_4^*} \tag{61}$$

where $\omega_i = k_i v_i$, $\omega_i^* = k_i^* v_i^*$, $(i = 1, 2, 3, 4)$.

For incident rotational waves,

$$\theta_3 = \theta_0, \quad v_0 = v_3. \tag{62}$$

For incident dilatational waves,

$$\theta_1 = \theta_0, \quad v_0 = v_1. \tag{63}$$

Using the boundary conditions, Eqs (51)–(58), we get a set of eight equations;

$$\sum_{j=1}^{8} a_{ij} zj = b_i, \quad (i = 1, 2, \ldots, 8) \tag{64}$$

where

$$z1=\frac{E_1}{F_1}, \quad z2=\frac{E_2}{F_1}, \quad z3=\frac{F_3}{F_1}, \quad z4=\frac{F_4}{F_1}, \quad z5=\frac{E_1^*}{F_1}, \quad z6=\frac{E_2^*}{F_1}, \quad z7=\frac{F_3^*}{F_1}, \quad z8=\frac{F_4^*}{F_1} \tag{65}$$

and

$$a_{11} = -\left(\delta_1 k_1^2 + \delta_3 k_1^2 \sin^2\theta_1 + \gamma_1 \bar{\tau}_1 \beta_1\right),$$
$$a_{12} = -\left(\delta_1 k_2^2 + \delta_3 k_2^2 \sin^2\theta_2 + \gamma_1 \bar{\tau}_1 \beta_2\right),$$
$$a_{13} = \delta_3 k_3^2 \sin 2\theta_3/2, \quad a_{14} = \delta_3 k_4^2 \sin 2\theta_4/2,$$
$$a_{15} = \delta_1^* k_1^{*2} + \delta_3^* k_1^{*2} \sin^2\theta_1^* + \gamma_1^* \bar{\tau}_1^* \beta_1^*, \quad a_{16} = \delta_1^* k_2^{*2} + \delta_3^* k_2^{*2} \sin^2\theta_2^* + \gamma_1^* \bar{\tau}_1^* \beta_2^*,$$
$$a_{17} = \delta_3^* k_3^{*2} \sin 2\theta_3^*/2, \quad a_{18} = \delta_3^* k_4^{*2} \sin 2\theta_4^*/2,$$

$$
\begin{aligned}
&a_{21}=-\delta_3 k_1^2 \sin 2\theta_1/2, \quad a_{22}=-\delta_3 k_2^2 \sin 2\theta_2/2\\
&a_{23}=-k_3^2\left(\delta_5 \sin^2\theta_3-\delta_4\cos^2\theta_3\right)-k\beta_3,\\
&a_{24}=-k_4^2\left(\delta_5 \sin^2\theta_4-\delta_4\cos^2\theta_4\right)-k\beta_4,\\
&a_{25}=-\delta_3^* k_1^{*2}\sin 2\theta_1^*/2, \quad a_{26}=-\delta_3^* k_2^{*2}\sin 2\theta_2^*/2,\\
&a_{27}=k_3^{*2}\left(\delta_5^*\sin^2\theta_3^*-\delta_4^*\cos^2\theta_3^*\right)+k^*\beta_3^*,\\
&a_{28}=k_4^{*2}\left(\delta_5^*\sin^2\theta_4^*-\delta_4^*\cos^2\theta_4^*\right)+k^*\beta_4^*,\\
&a_{31}=a_{32}=0, \quad a_{33}=\gamma\beta_3 k_3\sin\theta_3, \quad a_{34}=\gamma\beta_4 k_4\sin\theta_4, \quad a_{35}=a_{36}=0\\
&a_{37}=\gamma^*\beta_3^* k_3^*\sin\theta_3^*, \quad a_{38}=\gamma^*\beta_4^* k_4^*\sin\theta_4^*\\
&a_{41}=k_1\cos\theta_1, \quad a_{42}=k_2\cos\theta_2, \quad a_{43}=-k_3\sin\theta_3, \quad a_{44}=-k_4\sin\theta_4,\\
&a_{45}=-k_1^*\cos\theta_1^*, \quad a_{46}=-k_2^*\cos\theta_2^*, \quad a_{47}=-k_3^*\sin\theta_3^*, \quad a_{48}=-k_4^*\sin\theta_4^*\\
&a_{51}=k_1\sin\theta_1, \quad a_{52}=k_2\sin\theta_2, \quad a_{53}=k_3\cos\theta_3, \quad a_{54}=k_4\cos\theta_4,\\
&a_{55}=k_1^*\sin\theta_1^*, \quad a_{56}=k_2^*\sin\theta_2^*, \quad a_{57}=-k_3^*\cos\theta_3^*, \quad a_{58}=-k_4^*\cos\theta_4^*,\\
&a_{61}=a_{62}=0, \quad a_{63}=\beta_3, \quad a_{64}=\beta_4, \quad a_{65}=a_{66}=0, \quad a_{67}=-\beta_3^*, \quad a_{68}=-\beta_4^*\\
&a_{71}=\beta_1, \quad a_{72}=\beta_2, \quad a_{73}=a_{74}=0, \quad a_{75}=-\beta_1^*, \quad a_{76}=-\beta_2^*, \quad a_{77}=a_{78}=0\\
&a_{81}=k'\beta_1 k_1\sin\theta_1, \quad a_{82}=k'\beta_2 k_2\sin\theta_2, \quad a_{83}=a_{84}=0,\\
&a_{85}=k'^*\beta_1^* k_1^*\sin\theta_1^*, \quad a_{86}=k'^*\beta_2^* k_2^*\sin\theta_2^*, \quad a_{87}=a_{88}=0
\end{aligned}
$$

For incident rotational waves:

$$b_1=a_{13},\ b_2=-a_{23},\ b_3=a_{33},\ b_4=a_{43},\ b_5=-a_{53},\ b_6=-a_{63},\ b_7=b_8=0 \tag{66}$$

For incident dilatational waves:

$$b_1=-a_{11},\ b_2=a_{21},\ b_3=0,\ b_4=-a_{41},\ b_5=a_{51},\ b_6=0,\ b_7=-a_{71},\ b_8=a_{81}, \tag{67}$$

where

$$
\begin{aligned}
&\delta_1=K+\frac{i\omega\overline{R}}{3}, \quad \delta_2=K+k-\frac{2i\omega\overline{R}}{3}, \quad \delta_3=k-i\omega\overline{R}, \quad \delta_4=k-\frac{i\omega\overline{R}}{2},\\
&\delta_5=-\frac{i\omega\overline{R}}{2}, \quad \delta_1^*=K^*+\frac{i\omega\overline{R}^*}{3}, \quad \delta_2^*=K^*+k^*-\frac{2i\omega\overline{R}^*}{3},\\
&\delta_3^*=k^*-i\omega\overline{R}^*, \quad \delta_4^*=k^*-\frac{i\omega\overline{R}^*}{2}, \quad \delta_5^*=-\frac{i\omega\overline{R}^*}{2}
\end{aligned} \tag{68}
$$

6. SPECIAL CASES

6.1. Neglecting the Micropolar Effect

In this case $k=\alpha=\beta=\gamma=k^*=\alpha^*=\beta^*=\gamma^*=0$, and there exists only three reflected waves and three refracted waves. Two of the three waves are dilatational waves, and the other one is a rotational wave. The equation for the displacement potentials, ϕ and ψ, are given below:

$$\left(\nabla^4+N_1\nabla^2+N_2\right)\phi_1=0 \tag{69}$$

$$\left(\nabla^2+N_3\right)\psi_1=0 \tag{70}$$

in which

$$\begin{aligned} &N_1=\frac{k'\alpha_1\omega^2-\xi_1\xi_3-\xi_2\xi_4}{k'\xi_1}, \quad N_2=-\frac{\alpha_1\omega^2\xi_3}{k'\xi_1}, \quad N_3=\frac{2\alpha_1\rho\omega}{i\overline{R}}, \\ &\xi_1=\frac{K+\mu_0H_0^2+2/3i\omega\overline{R}}{\rho}, \quad \gamma_1=(3\lambda+2\mu)\alpha_t, \end{aligned} \tag{71}$$

The velocities of the rotational waves are given as

$$v_{1,2}^2=\frac{\omega^2\left(N_1\pm\sqrt{N_1^2-4N_2}\right)}{2N_2}; \tag{72}$$

the velocity of the dilatational wave is

$$v_3^2=\frac{\omega^2}{N_3} \tag{73}$$

6.2. Neglecting the Viscous Effect

Here the media M_1 and M_2 are micropolar and elastic. In this case $R=2\mu$ and

$$\begin{aligned} &N_1=\frac{k'\alpha_1\omega^2-\xi_1\xi_3-\xi_2\xi_4}{k'\xi_1}, \quad N_2=-\frac{\alpha_1\omega^2\xi_3}{k'\xi_1}, \quad \xi_1=\frac{\lambda+2\mu+k+\mu_0H_0^2}{\rho}, \\ &N_3=\frac{\left(\rho J\omega^2-2k\right)\mu+\gamma\alpha_1\rho\omega^2+k^2}{\mu\gamma}, \quad N_4=\frac{\alpha_1\rho\omega^2\left(\rho J\omega^2-2k\right)}{\mu\gamma}, \\ &\beta_j=\frac{\alpha_1\rho\omega_j^2-\mu k_j^2}{k}, \quad (j=3,4) \end{aligned} \tag{74}$$

The expressions for the velocities of the rotational and dilatational waves are the same as Eqs (36) and (37). Also,

$$\begin{aligned} &\delta_1=\lambda, \quad \delta_2=\lambda+\mathrm{k}+2\mu, \quad \delta_3=\mathrm{k}+2\mu, \quad \delta_4=\mathrm{k}+\mu, \\ &\delta_5=\mu, \quad \delta_1^*=\lambda^*, \quad \delta_2^*=\lambda^*+\mathrm{k}^*+2\mu^*, \\ &\delta_3^*=\mathrm{k}^*+2\mu^*, \quad \delta_4^*=\mathrm{k}^*+\mu^*, \quad \delta_5^*=\mu^*. \end{aligned} \tag{75}$$

7. NUMERICAL RESULTS

According to Refs 21 and 22, and assuming medium M_1 is an aluminum-epoxy micropolar viscoelastic material, and medium M_2 is a magnesium crystal micropolar viscoelastic material, the parameters are given below

For Medium M_1 [21]:

$$\begin{aligned} \lambda &= 7.59\times 10^{10}\,\mathrm{Pa}, \mu=1.89\times 10^{10}\,\mathrm{Pa}, k=0.0149\times 10^{10}\,\mathrm{Pa}, \\ \rho &= 2.19\,\mathrm{g\cdot cm^{-3}}, J=0.0196\,\mathrm{cm}^2, \\ k' &= 0.48\ \mathrm{cal\cdot cm^{-1}\cdot s^{-1}\cdot {}^\circ C^{-1}}, C_\mathrm{E}=0.206\mathrm{cal\cdot g^{-1}\cdot {}^\circ C^{-1}}, \\ \alpha_\mathrm{t} &= 2.35\times 10^{-5\circ}\mathrm{C}^{-1}, \gamma=2.68\times 10^9\mathrm{g\cdot cm\cdot s^{-2}} \\ \tau_0 &= 3.13\times 10^{-12}\mathrm{s},\ \ \mathrm{and}\tau_1=4.575\times 10^{-12}\mathrm{s}. \end{aligned}$$

For Medium M_2 [22]:

$$\begin{aligned} \lambda^* &= 9.4\times 10^{10}\ \mathrm{Pa}\ , \mu^*=4.0\times 10^{10}\ \mathrm{Pa}\ , k^*=1.0\times 10^{10}\ \mathrm{Pa}\ , \\ \rho^* &= 1.74\,\mathrm{g\cdot cm^{-3}}, J^*=0.2\times 10^{-15}\mathrm{cm}^2, \\ k'^* &= 0.06\ \mathrm{cal\ \cdot cm^{-1}\cdot s^{-1}\cdot {}^\circ C^{-1}}, C_\mathrm{E}^*=0.23\ \mathrm{cal\ \cdot g^{-1}\cdot {}^\circ C^{-1}}, \\ \alpha_\mathrm{t}^* &= 7.4033\times 10^{-6\circ}\mathrm{C}^{-1}, \gamma=7.79\times 10^{-5}\mathrm{g\cdot cm\cdot s^{-2}} \\ \tau_0^* &= 1.565\times 10^{-11}\ \mathrm{s},\ \ \mathrm{and}\ \tau_1^*=2.2875\times 10^{-11}\mathrm{s}. \end{aligned}$$

Suppose the initial temperature for the two media is $T_0 = 300\,\mathrm{K}$ and $\omega^2/\omega_0^2 = 10$. Dimensionless parameters in the constitutive equations for two viscoelastic materials are taken as [20] $\beta = 0.005$, $A = 0.106$, and $\alpha' = 0.5$.

Figures 2 and 3 give the variation of the reflection and refraction coefficient ratios with the angle of incidence for the rotational and dilatational waves based on the three theories. We can see that in the case of an incident rotational wave, the reflection and refraction coefficient ratios $|z1| = |z2| = |z4| = |z5| = |z6| = |z7| = |z8| = 0$ and $|z3| = 1$ when $\theta = 0^0$, and $|z1| = |z2| = |z5| = |z6| = 0$ when $\theta = 90^0$. Also, we can observe that there

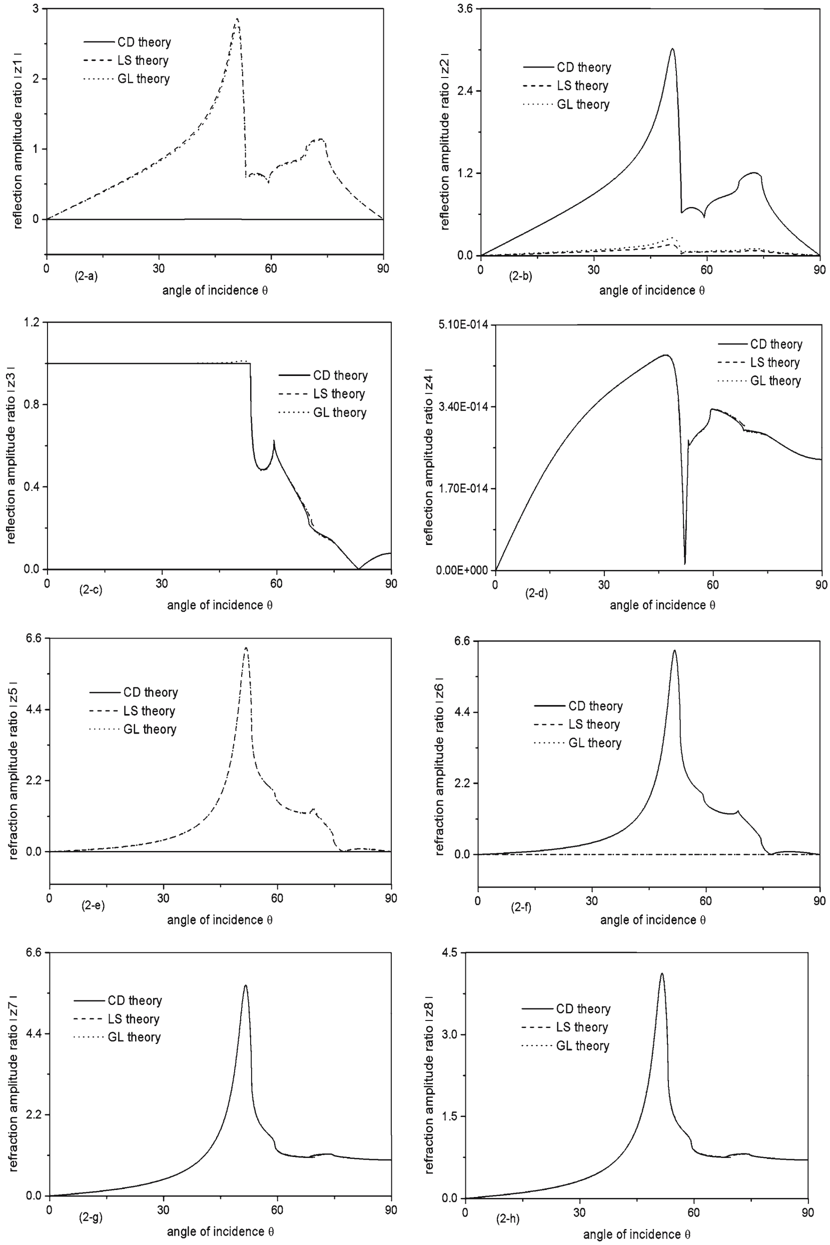

Fig. 2. Variation of reflection and refraction coefficient ratios with incident angle of rotational wave under various theories.

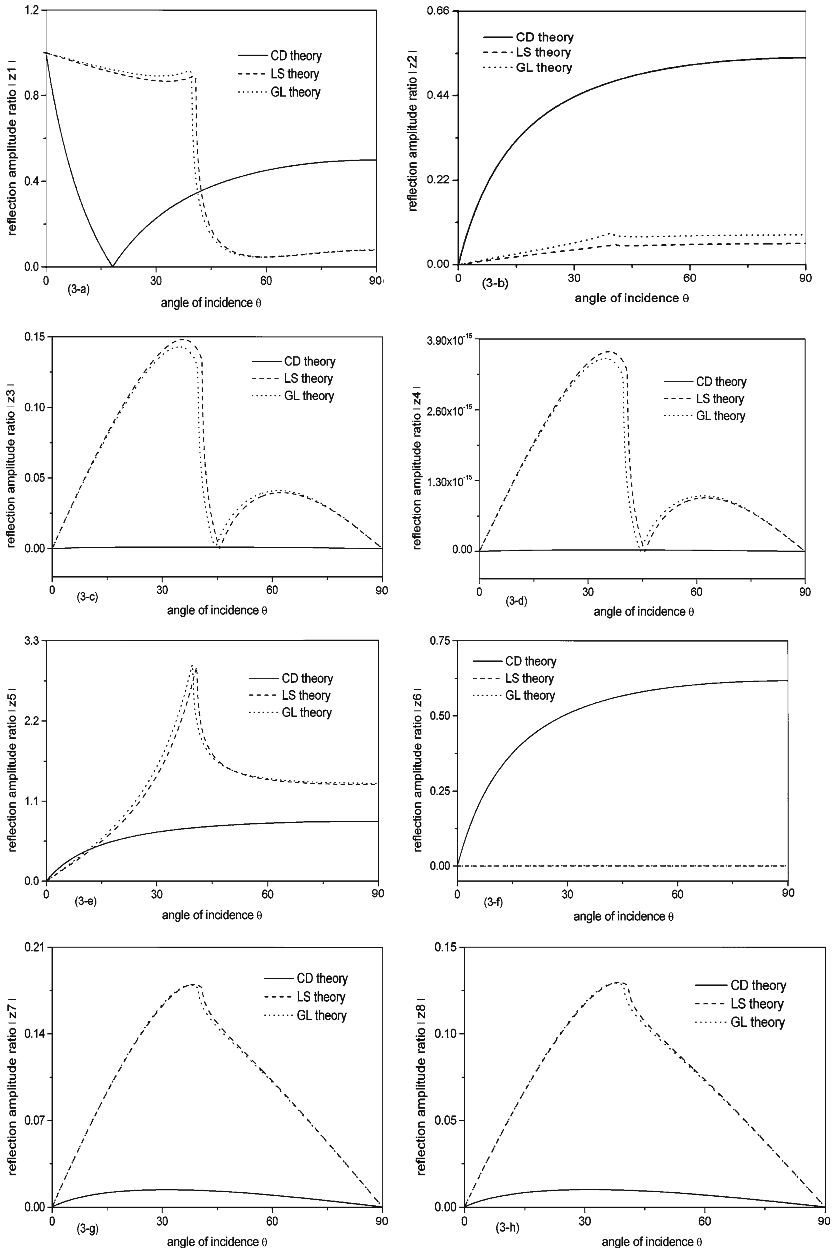

Fig. 3. Variation of reflection and refraction coefficient ratios with incident angle of rotational wave under various theories.

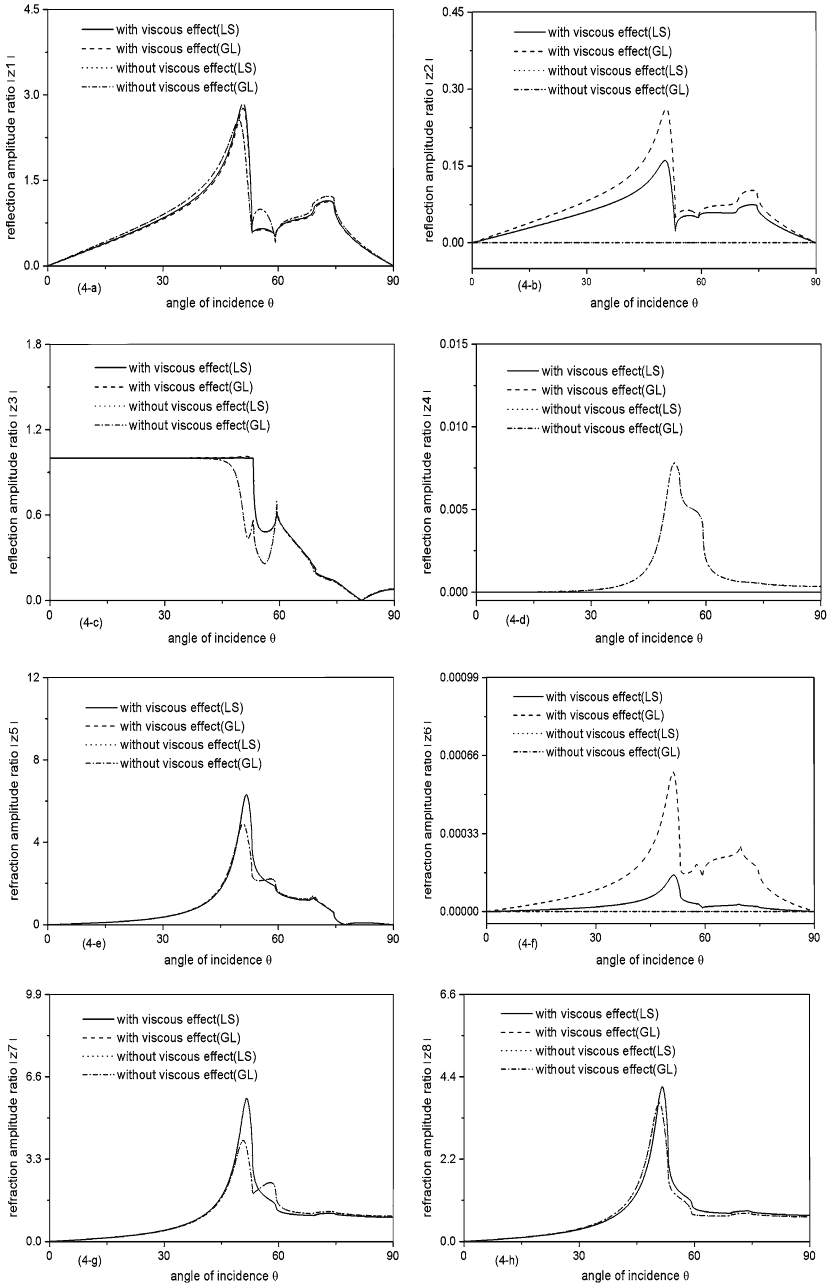

Fig. 4. Viscous effect on variation of reflection and refraction coefficient ratios with incident angle of rotational wave.

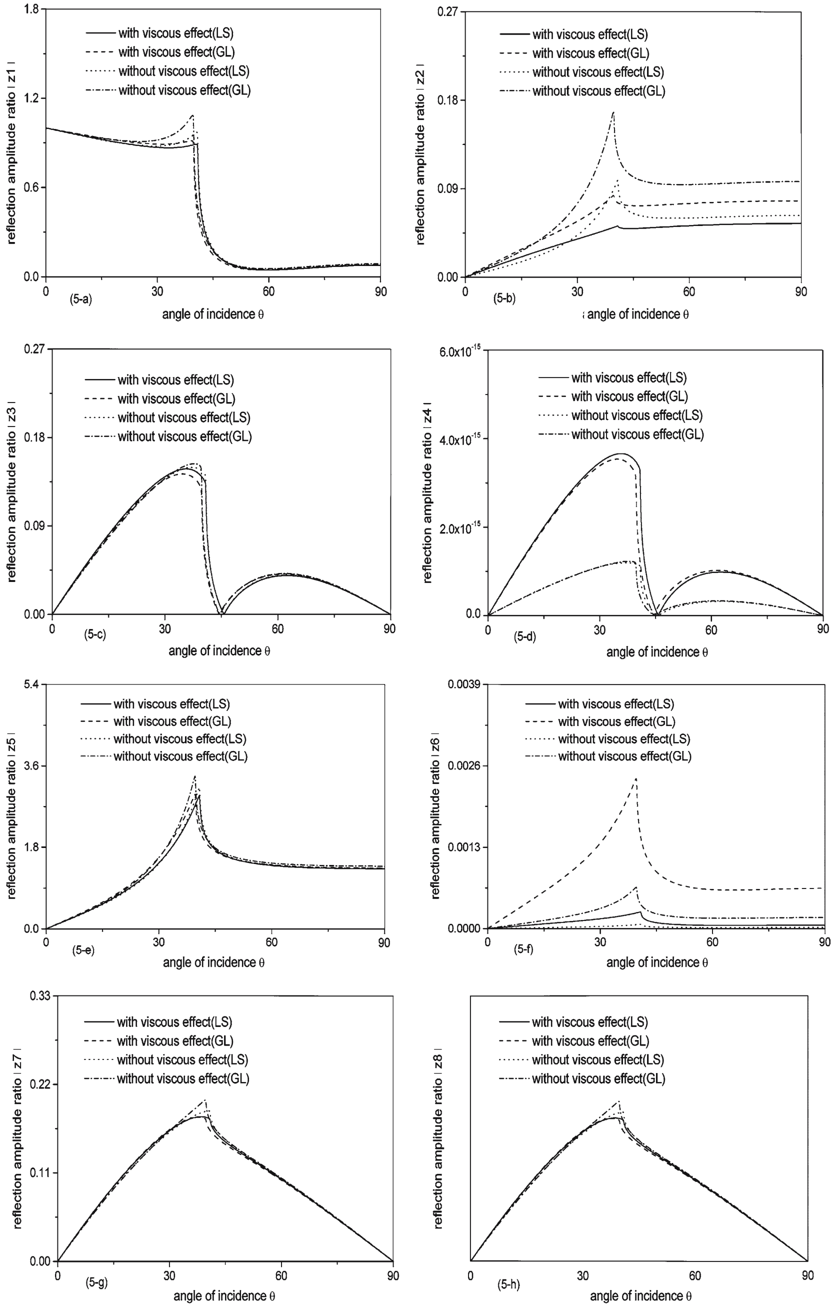

Fig. 5. Viscous effect on variation of reflection and refraction coefficient ratios with incident angle of dilatational wave.

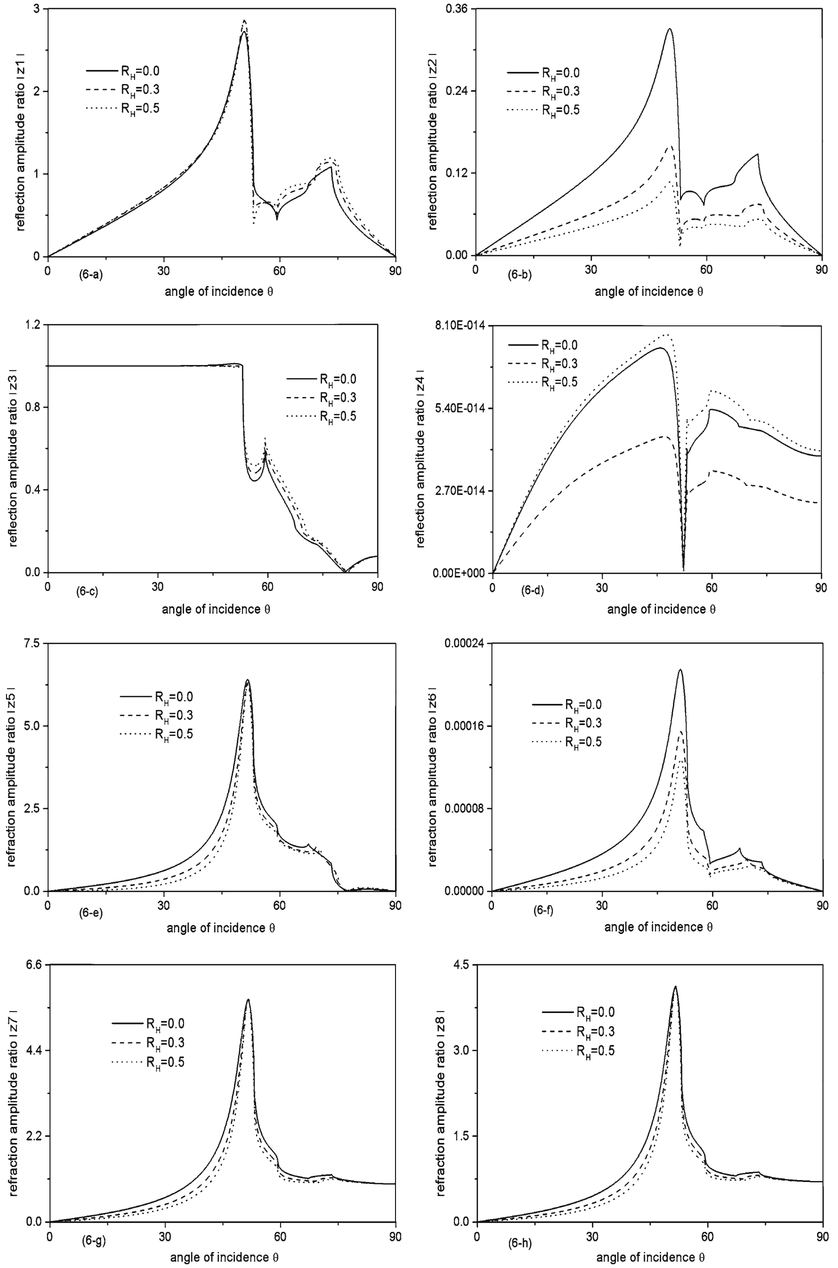

Fig. 6. Effect of magnetic field on variation of reflection and refraction coefficient ratios with incident angle of rotational wave.

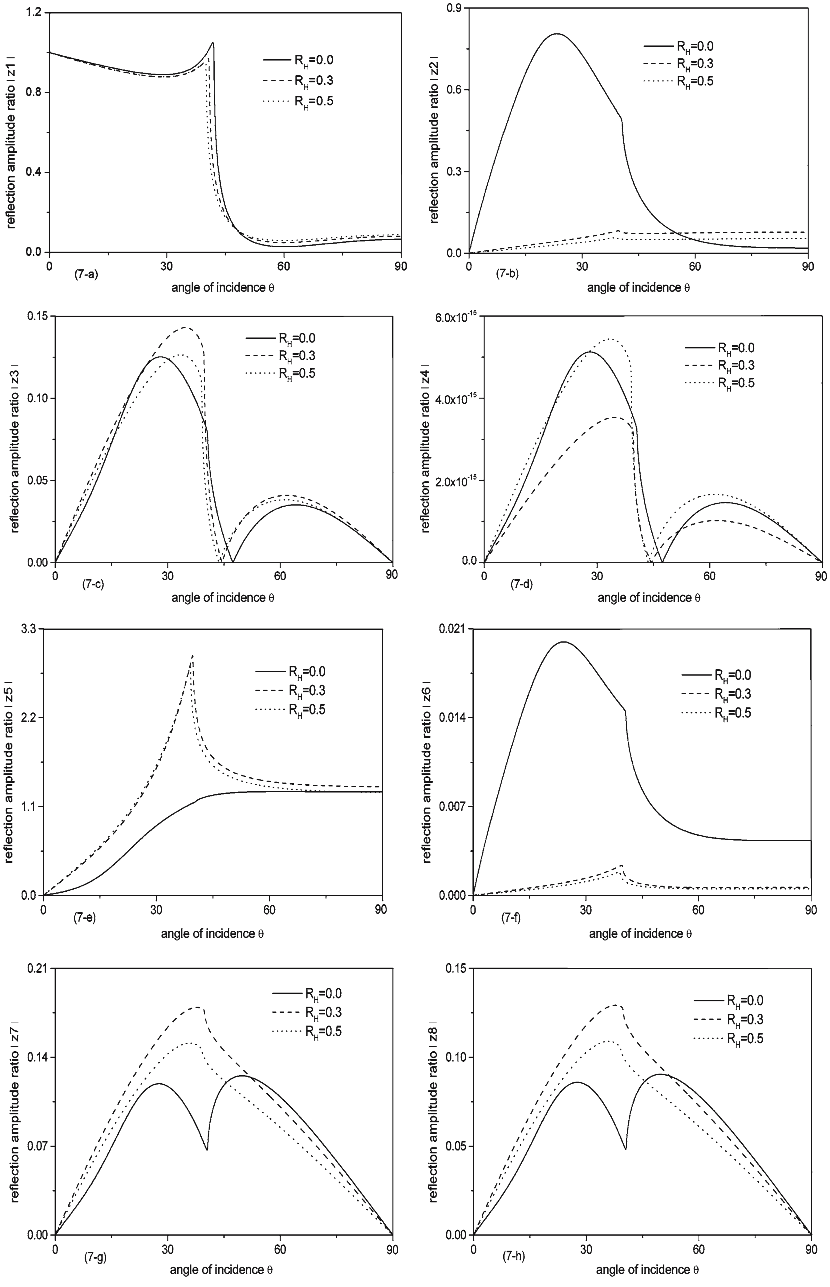

Fig. 7. Effect of magnetic field on variation of reflection and refraction coefficient ratios with incident angle of dilatational wave.

are significant differences under generalized theories and conventional theory for $|zi|$, $(i=1,2,5,6)$. The differences are very small under CD, GL, and LS theories for $|zi|$, $(i=3,4,7,8)$. For the case of an incident dilatational wave, the reflection and refraction coefficient ratios $|z2|=|z3|=|z4|=|z5|=|z6|=|z7|=|z8|=0$ and $|z1|=1$ when $\theta=0^0$, and $|z3|=|z4| = |z7|=|z8|=0$ when $\theta=90^0$. We also can see there is an obvious difference under generalized and conventional theories for $|zi|$, $(i=1,2,\ldots.8)$, and the difference is very small under GL and LS theories. Figures 4 and 5 show the viscous effect under LS and GL theories for two different cases. It can be observed that the viscous effect plays an important role. For many materials the viscous effects have been neglected to simplify calculations. However, from computational results in this paper, we can see that we will obtain incorrect conclusions if we neglect the viscous effects. Figures 6 and 7 show the variation of the angle of incidence with the reflection coefficient ratios for different values of the magnetic field under the GL theory. Clearly the magnetic field has a salient influence on the reflection and refraction coefficient ratios. Thus, the magnetic field can be used to change the angle and values of the reflected and refracted waves.

8. CONCLUSIONS

We obtain the following conclusions according to the above analysis:

1. The reflection and refraction coefficient ratios depend on the angle of incidence, and the nature of this dependence is different for different reflected waves.
2. The viscous effects play a significant role.
3. The magnetic field has a salient influence on the reflection and refraction coefficient ratios.

ACKNOWLEDGMENTS

This work was supported by grants (No.10502041) from the National Natural Science Foundation of PRC and the Natural Foundation of Xi'an Jiaotong University.

NOMENCLATURE

$\lambda, \mu, k, \alpha, \beta, \gamma$	module of the medium
C_E	specific heat at constant strain
T	absolute temperature
σ_{ij}	components of stress tensor

ρ	density
t	time
T_0	reference temperature
ε_{ij}	components of strain tensor
m_{ij}	couple stress
u_i	components of displacement vector
μ_0	magnetic permeability
τ_0, τ_1	relaxation time
e	dilatation
γ	$(3\lambda + 2\mu + k)\,\alpha_t$
$\vec{H}$	initial uniform magnetic intensity vector
$\vec{\mathbf{h}}$	induced magnetic field vector vector
$\vec{\mathbf{J}}$	current density vector
	bulk modulus
ε	coupling parameter
A, β, α'	empirical constants
ω_i	components of microrotation vector
k'	thermal conductivity
ε_0	dielectric constant
J	micro inertia moment
α_t	coefficient of linear thermal expansion
ϕ, ψ	displacement potential
$\vec{\mathbf{E}}$	induced electric field vector
$\vec{\mathbf{u}}$	displacement
K	$\lambda + (2/3)\,\mu$
R (t)	relaxation function
n	dimensionless constant

REFERENCES

1. H. W. Lord and Y. Shulman, *J. Mech. Phys. Solids* **15**:299 (1967).
2. A. E. Green and K. E. Lindsay, *J. Elasticity* **2**:1 (1972).
3. D. S. Chandrasekharaiah, *Appl. Mech. Rev*. **39**:355 (1986).
4. D. S. Chandrasekharaiah, *Appl. Mech. Rev*. **51**:705 (1998).
5. A. C. Eringen, *Theory of Micropolar Elasticity*, in *Fracture*, Vol. 2, H. Leibowitz, ed. (Academic Press, New York, 1968), pp. 662–729.
6. W. Nowacki, *Bull. Acad. Polon. Sci., Ser. Sci. Tech*. **14**:129 (1966).
7. W. Nowacki, *Bull. Acad. Polon. Sci., Ser. Sci. Tech*. **14**:263 (1966).

8. W. Nowacki, *Bull. Acad. Polon. Sci., Ser. Sci. Tech*. **14**:801 (1966).
9. A. C. Eringen, *Int. J. Eng. Sci*. 4:179 (1966).
10. F. Passarella, *Mech. Res. Commun*. **23**:349 (1996).
11. S. Nath, P. R. Sengupta, and L. Debnath, *Computers Math. Applic*. **35**:47 (1998).
12. A. S. EI-Karamany, *Int. J. Eng. Sci.*, **40**:2097 (2002).
13. Y.-Q. Song, Y.-C. Zhang, H.-Y. Xu, and B.-H. Lu, *Int. J. Thermophys*. **25**:909 (2004).
14. V. R. Parfitt and A. C. Eringen, *J. Acoust. Soc. Amer*. **45**:1258 (1969).
15. T. Ariman, *Acta Mechanica* **13**:11 (1972).
16. B. Singh and R. Kumar, *Int. J. Eng. Sci*. **36**:101 (1998).
17. B. Singh and R. Kumar, *Int. J. Eng. Sci*. **36**:119 (1998).
18. R. Kumar, *Int. J. Eng. Sci*. **38**:1377 (2000).
19. R. Kumar and S. Deswal, *J. Sound Vibration* **256**:173 (2002).
20. M. A. Ezzat, M. I. Othman, and A. S. EI-Karamany, *Int. J. Eng. Sci*. **40**:283 (2002).
21. R. Kumar and S. Deswal, *J. Sound Vibration* **239**:467 (2001).
22. B. Singh, *J. Sound Vibration* **235**:685 (2000).